曾鸣聪

玉泉数学中心 203

March 3, 2012, Saturday, 13:30-16:30

Title On division algebras and parallelizable spheres

Abstract In 1958, R. Bott and J. Milnor proved that only the dimension 1,3,7 spheres are parallelizable and the division algebra structures only exists on dimension 1,2,4,8 real vector spaces by using Pontrjagin classes. After that, J. F. Adams give another proof in 1960 by showing the non-existance of map of Hopf invariant one for maps from S^{4n-1} to S^{2n}. This proof is quite difficult and long (the original paper is about 20 pages). But after the invention of topological K-theory by M. Atiyah and F. Hirzebruch one can reduce the length of the proof since K-theory has some amazingly good property such as Bott periodicity and splitting principle.

In this talk I will give a proof of the existence of parallelizable spheres only in dimensions 1,3,7 and real division algebras only in dimensions 1,2,4,8 by introducing complex K-theory. Some long proof such as Bott periodicity and splitting principle will be omitted. Because some are not familiar with vector bundles I shall give a brief introduction of vector bundles first. A good introduction on vector bundles can be found in [2] and the definition and properties of cohomology theory can be found in [3].

References

[1] *K-Theory*, M. Atiyah

[2] *Vector Bundles and K-Theory*, A. Hatcher (Available in his homepage)

[3] *Algebraic Topology*, A. Hatcher